

Symmetric categorial grammar

Wednesday

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Contents

1	The plan for today	4
2	In a picture	5
3	Proofs and Refutations	6
4	Summary of the steps: Exercise	7
4.1	Solution: Sequent	8
4.2	Trees vs. Co-trees	12
4.3	Solution: Lowering	13
5	Connection to Montague interpretation	14
5.1	Mappings: when	15
6	How to solve for $\llbracket \cdot \rrbracket$ and $\llbracket \cdot \rrbracket_{\perp}$	16
6.1	Some combinators	17
6.2	Mappings: example	18
6.3	Constants: CBV vs. CBN	19
6.4	Constants	20
6.5	Montagovian connection: example	21
7	Scope and binding: Test suite	22
7.1	Simple QP Context: Someone left	23

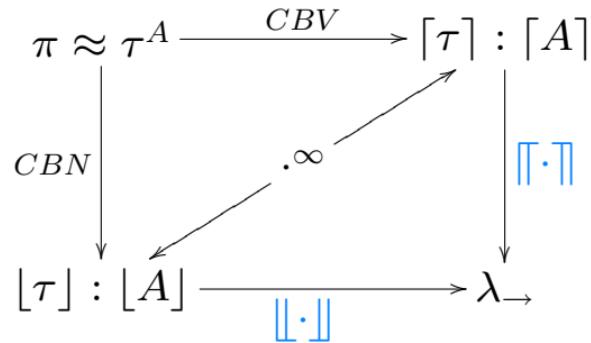


7.2	Lifting for \exists, \forall	24
7.3	Lifting for \exists, \forall : call-by-name	25
7.4	Connection to Montague interpretations: CPS	26
8	Scope ambiguity, local	27
8.1	Ambiguity: CPS in CBN	30
9	Step by step	31
9.1	What if	33
10	What have you learned today	34

1. The plan for today

- ▶ Recap: proofs and refutations
- ▶ Solutions to some exercises
- ▶ Connecting lexical and derivational semantics
- ▶ Scope construal test suite, with analyses satisfying
 - ▷ type uniformity
 - ▷ scope flexibility

2. In a picture



3. Proofs and Refutations

Proof: A implies B

$$A \vdash_{\text{CBV}^{\triangleright}} \textcolor{magenta}{B}$$



$$\textcolor{magenta}{A} \vdash_{\text{CBV}^{\triangleleft}} B$$

$$V_A \rightarrow C_B$$

$$K_B \rightarrow K_A$$

$$(V_A \times K_B) \rightarrow R = [A \setminus B] \quad \bowtie \quad [B/A] = (K_B \times V_A) \rightarrow R$$

Refutation: A and $\neg B$

$$\textcolor{magenta}{A} \vdash_{\text{CBN}^{\triangleleft}} B$$



$$A \vdash_{\text{CBN}^{\triangleright}} \textcolor{magenta}{B}$$

 ∞ ∞

$$B^\infty \vdash_{\text{CBV}^{\triangleright}} \textcolor{magenta}{A}^\infty$$

$$\textcolor{magenta}{B}^\infty \vdash_{\text{CBV}^{\triangleleft}} A^\infty$$

$$K_B \rightarrow (C_A \rightarrow R)$$

$$C_A \rightarrow C_B$$

$$(K_B \times C_A) \rightarrow R = [A \oslash B] \quad \bowtie \quad [B \oslash A] = (C_A \times K_B) \rightarrow R$$

4. Summary of the steps: Exercise

“Alice teases Lewis”

- ▶ Sequent
- ▶ Labelled Sequent
- ▶ CPS interpretation: CBV and CBN
- ▶ CBN $^\triangleright$ read as CBV $^\triangleleft$

4.1. Solution: Sequent

$$\frac{\frac{\frac{np \longrightarrow np \quad s \longrightarrow s}{np \circ np \setminus s \longrightarrow s}}{np \circ ((np \setminus s) / np \circ np) \longrightarrow s} \quad np \circ ((np \setminus s) / np \circ np) \longrightarrow s}{np \circ ((np \setminus s) / np \circ np) \longrightarrow s}$$

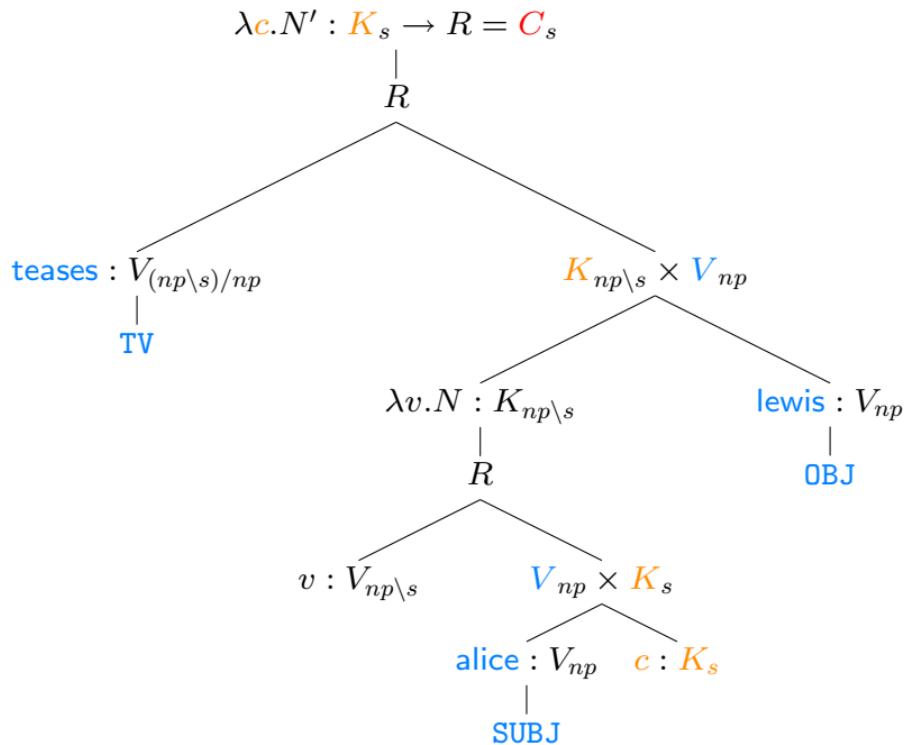
Labelled Sequent

$$\frac{\frac{\frac{x : np \xrightarrow{x} np \quad s \xrightarrow{\alpha} \alpha : s}{x : np \circ np \setminus s \xrightarrow{x \bowtie \alpha} \alpha : s}}{x : np \circ ((np \setminus s) / np \circ y : np) \xrightarrow{(x \bowtie \alpha) \bowtie y} \alpha : s} \quad y : np \xrightarrow{y} np}{x : np \circ (z : (np \setminus s) / np \circ y : np) \xrightarrow{\mu\alpha.(z * ((x \bowtie \alpha) \bowtie y))} s}$$

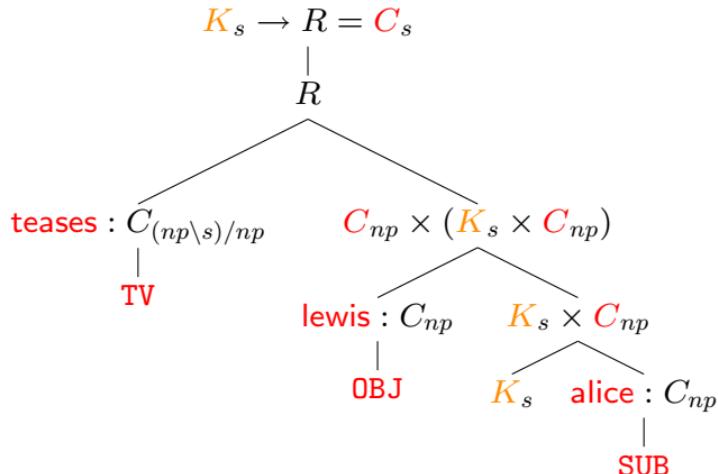
By replacing the variables with the corresponding lexical constants:

$$\mu\alpha.(\text{teases} * ((\text{alice} \bowtie \alpha) \bowtie \text{lewis})) (= M)$$

$\text{CBV}^\triangleright : \lceil M \rceil = \lambda c. (\text{teases} \langle \lambda v. (v \langle \text{alice}, c \rangle), \text{lewis} \rangle) :$



$\text{CBN}^\triangleright \lfloor M \rfloor = \lambda c.(\text{teases } \langle \text{lewis}, \langle c, \text{alice} \rangle \rangle).$



4.2. Trees vs. Co-trees

$$A \vdash_{\text{CBV}^\triangleright} B \quad V_A \rightarrow C_B$$

$$A \vdash_{\text{CBN}^\triangleright} B \quad C_A \rightarrow C_B$$

$$A \vdash_{\text{CBV}^\triangleright} B \quad \infty \ A \vdash_{\text{CBN}^\triangleleft} B \quad = \quad B^\infty \vdash_{\text{CBV}^\triangleright} A^\infty$$

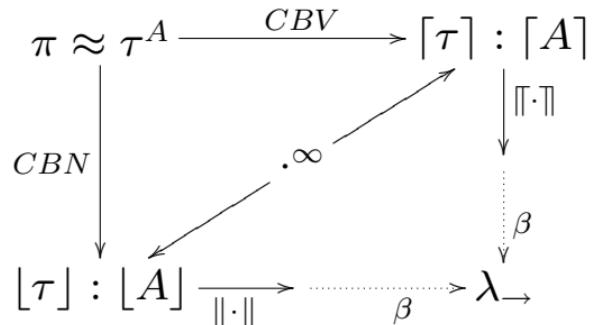
$$A \vdash_{\text{CBV}^\triangleleft} B \quad \infty \ A \vdash_{\text{CBN}^\triangleright} B \quad = \quad B^\infty \vdash_{\text{CBV}^\triangleleft} A^\infty$$

4.3. Solution: Lowering

$$\frac{\frac{\frac{y : s \xrightarrow{y} s \quad np \xrightarrow{\alpha} \alpha : np}{s \xrightarrow{y \succ \alpha} (s \oslash np) \circ np}}{\frac{s \xrightarrow{\tilde{\mu}y.((y \succ \alpha) * \beta)} \beta : (s \oslash np) \circ np}{(s \oslash np) \oslash s \xrightarrow{\beta \oslash (\tilde{\mu}y.((y \succ \alpha) * \beta))} \alpha : np}}{z : (s \oslash np) \oslash s \xrightarrow{\mu\alpha.(z * (\beta \oslash (\tilde{\mu}y.((y \succ \alpha) * \beta)))))} np}$$

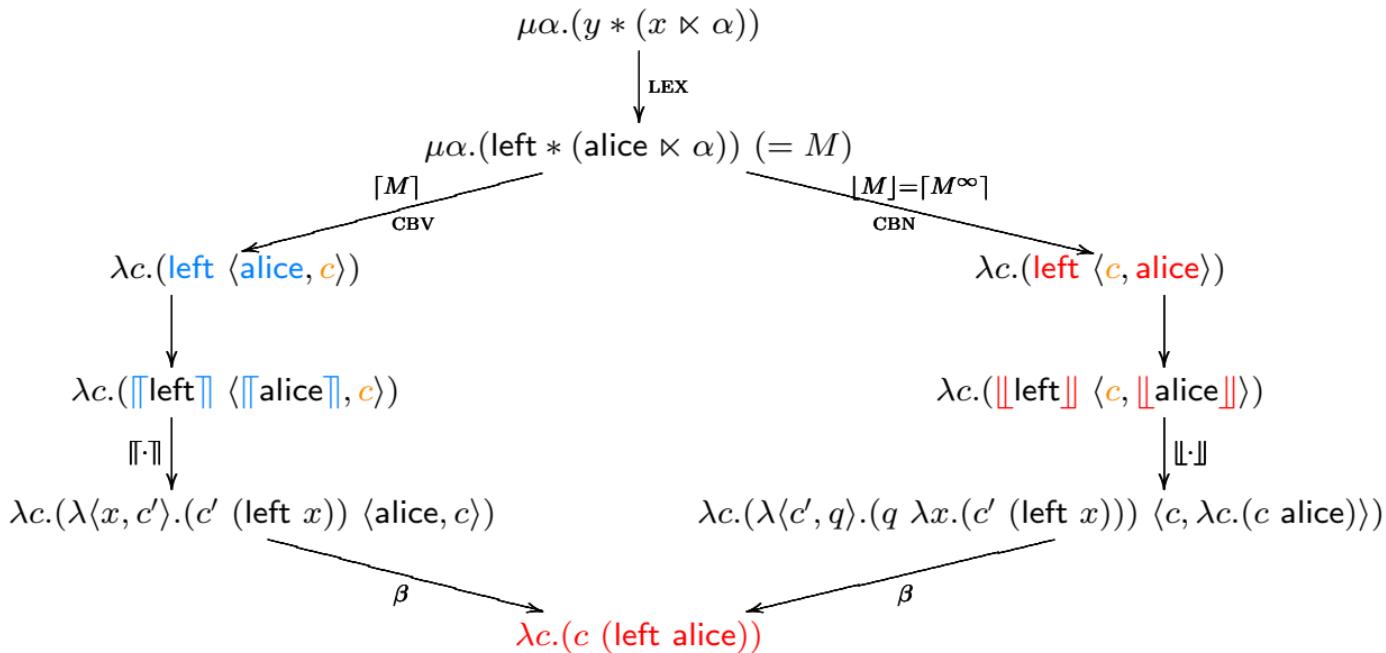
$$\mu\alpha.(z * (\beta \oslash (\tilde{\mu}y.((y \succ \alpha) * \beta))))$$

5. Connection to Montague interpretation



- ▶ $\llbracket \cdot \rrbracket$: Constants to **Values**
- ▶ $\llbracket \cdot \rrbracket$: Constants to **Computations**

5.1. Mappings: when



6. How to solve for $\llbracket \cdot \rrbracket$ and $\llbracket \cdot \rrbracket$

Source language In a Montague-style interpretation, a lexical item w of syntactic type A is interpreted by a closed term ϕ of semantic type A' , with

$$np' = e, \quad s' = t, \quad (A \setminus B)' = (B/A)' = A' \rightarrow B'$$

Target language To lift the interpretation to the CPS level, we want to find a solution for the following equations (identifying R and t):

$$\begin{array}{ll} \text{cbv} & \phi_{A'} \xrightarrow[\lambda]{M} \lceil A \rceil \\ \text{cbn} & \phi_{A'} \xrightarrow[\lambda]{N} \lfloor A \rfloor \end{array}$$

where M, N are the terms you obtain for IL proofs of the type transitions from A' to $\lceil A \rceil$ or $\lfloor A \rfloor$.

Which Logic? Full IL? Linearity condition ($= \mathbf{LP}^*$)? Allow closed terms? (yes)

6.1. Some combinators

Terms for proofs of the form $\vdash A$ are **pure** lambda terms, or combinators: no constants. Some examples.

- $\vdash A \rightarrow A$ ID
 $\lambda x.x$
- $\vdash A \rightarrow ((A \rightarrow B) \rightarrow B)$ LIFT
 $\lambda x\lambda y.(y\ x)$
- $\vdash (A \rightarrow A \rightarrow B) \rightarrow (A \rightarrow B)$ REFL
 $\lambda x\lambda y.((x\ y)\ y)$
- $\vdash ((A \rightarrow A) \rightarrow A) \rightarrow ((A \rightarrow A) \rightarrow A)$ RESET
 $\lambda x\lambda y.(y\ (x\ \text{ID}))$
- $\vdash ((A \rightarrow ((B \rightarrow B) \rightarrow B)) \rightarrow ((B \rightarrow B) \rightarrow B)) \rightarrow ((A \rightarrow B) \rightarrow B)$ SHIFT
 $\lambda x\lambda y.((x\ \lambda v\lambda z.(z\ (y\ v)))\ \text{ID})$
- ...

6.2. Mappings: example

Let's see:

$$\text{cbv} \quad \text{left}_{(np \setminus s)'} \xrightarrow{M_{\lambda \rightarrow}} \lceil np \setminus s \rceil$$

$$\text{cbn} \quad \text{left}_{(np \setminus s)'} \xrightarrow{N_{\lambda \rightarrow}} \lfloor np \setminus s \rfloor$$

6.3. Constants: CBV vs. CBN

We need to lift the constants of the Montagovian interpretation (e.g. `left`: $e \rightarrow t$, and `teases`: $(e \rightarrow (e \rightarrow t))$) to the CBV and CBN level.

Recall, for CBV, arguments are `values`, hence, $\llbracket \cdot \rrbracket$: Constants \rightarrow Values.

$$\llbracket \text{alice} \rrbracket = \text{alice}$$

$$\llbracket \text{left} \rrbracket = \lambda \langle x, c \rangle. (c (\text{left } x))$$

$$\llbracket \text{teases} \rrbracket = \lambda \langle v, y \rangle. (v \lambda \langle x, c \rangle. (c ((\text{teases } y) \ x)))$$

$$\llbracket \text{somebody} \rrbracket = \lambda \langle c, v \rangle. (\exists \lambda x. (v \langle x, c \rangle))$$

$$\llbracket np \rrbracket = np$$

$$\llbracket np \setminus s \rrbracket = R^{\llbracket np \rrbracket \times R^{\llbracket s \rrbracket}}$$

$$\llbracket iv / np \rrbracket = R^{R^{\llbracket iv \rrbracket} \times \llbracket np \rrbracket}$$

$$\llbracket s / (np \setminus s) \rrbracket = R^{R^{\llbracket s \rrbracket} \times \llbracket np \setminus s \rrbracket}$$

and for CBN, arguments are `computations`, hence, $\llbracket \cdot \rrbracket$: Constants \rightarrow Computations.

$$\llbracket \text{alice} \rrbracket = \lambda c. (c \text{ alice})$$

$$\llbracket \text{left} \rrbracket = \lambda \langle c, q \rangle. (q \lambda x. (c (\text{left } x)))$$

$$\llbracket \text{teases} \rrbracket = ?$$

$$C_{\llbracket np \setminus s \rrbracket} = R^{\llbracket s \rrbracket \times R^{\llbracket np \rrbracket}}$$

$$C_{\llbracket iv / np \rrbracket} = R^{R^{\llbracket np \rrbracket} \times \llbracket iv \rrbracket}$$

6.4. Constants

$$1. \llbracket \text{teases} \rrbracket = \lambda \langle q, \langle c, q' \rangle \rangle. (\underbrace{q'}_{C_{SUB}} \underbrace{\lambda x. (\underbrace{q}_{C_{OBJ}} \underbrace{\lambda y. (c ((\text{teases } y) x)))}_{K_{OBJ}})}_{K_{SUB}})$$

$$2. \llbracket \text{teases} \rrbracket = \lambda \langle q, \langle c, q' \rangle \rangle. (\underbrace{q}_{C_{OBJ}} \underbrace{\lambda y. (\underbrace{q'}_{C_{SUB}} \underbrace{\lambda x. (c ((\text{teases } y) x)))}_{K_{SUB}})}_{K_{OBJ}})$$

$$3. * \llbracket \text{teases} \rrbracket = \lambda \langle q, \langle c, q' \rangle \rangle. (\underbrace{q'}_{C_{SUB}} \underbrace{\lambda y. (\underbrace{q}_{C_{OBJ}} \underbrace{\lambda x. (c ((\text{teases } y) x)))}_{K_{SUB}})}_{K_{OBJ}})$$

$$4. * \llbracket \text{teases} \rrbracket = \lambda \langle q, \langle c, q' \rangle \rangle. (\underbrace{q}_{C_{OBJ}} \underbrace{\lambda x. (\underbrace{q'}_{C_{SUB}} \underbrace{\lambda y. (c ((\text{teases } y) x)))}_{K_{OBJ}})}_{K_{SUB}})$$

Note, 1. and 2. give the same readings when replaced to **teases** ... in a bit.

6.5. Montagovian connection: example

“Alice left”: $\mu\alpha.(\text{left} * (\text{alice} \ltimes \alpha))$

$$[M] = \lambda c.((\text{left } \langle \text{alice}, c \rangle))$$

$$\lfloor M \rfloor = \lambda c.((\text{left } \langle c, \text{alice} \rangle))$$

$$\begin{aligned} \lambda c.((\llbracket \text{left} \rrbracket \langle \llbracket \text{alice} \rrbracket, c \rangle)) &= \lambda c.(((\lambda \langle x, c' \rangle.(c' (\text{left } x))) \langle \text{alice}, c \rangle)) \\ &= \lambda c.(c (\text{left } \text{alice})) \end{aligned}$$

$$\begin{aligned} \lambda c.((\llbracket \text{left} \rrbracket \langle c, \llbracket \text{alice} \rrbracket \rangle)) &= \lambda c.(((\lambda \langle c', q \rangle.(q \lambda x.(c' (\text{left } x)))) \langle c, (\lambda c''.c'' \text{ alice}) \rangle) \\ &= \lambda c.((\lambda c''.c'' \text{ alice}) \lambda x.(c (\text{left } x))) \\ &= \lambda c.((\lambda x.(c (\text{left } x))) \text{ alice}) \\ &= \lambda c.(c (\text{left } \text{alice})) \end{aligned}$$

7. Scope and binding: Test suite

Claim LG gives us a way to express type assignment, $(s \oslash s) \odot np$ that overcomes the problems of a Lambek assignment $s/(np \setminus s)$.

- ▶ Simple QP context: $(s \oslash s) \odot np$ versus $s/(np \setminus s)$
 - ▷ (a) labelled sequent;
 - ▷ (b) CBV vs. CBN.
- ▶ Multiple QP examples:
 - ▷ scope ambiguity: local. (a) SUB wide scope OBJ wide scope. CBN
 - ▷ local versus non-local construal. (a) labelled sequent. (b) CBN

7.1. Simple QP Context: Someone left

$$\mu\beta.(z * (\gamma \odot (\tilde{\mu}x.(\mu\alpha.(y * (x \ltimes \alpha)) \succ \beta) * \gamma)))$$

1. $C_s : (\mu\alpha.(IV * (x \ltimes \alpha)) \succ \beta) (= M)$
2. $K_{np} : \tilde{\mu}x.(M * \gamma) (= K)$
3. $C_s : \mu\alpha.(QP * (\gamma \odot K))$

$$\begin{aligned}\lceil M \rceil &= \lambda c.((\text{left } \langle \pi^2 \text{someone}, \lambda z.(\pi^1 \text{someone } \langle z, c \rangle) \rangle)); \\ \lfloor M \rfloor &= \lambda c.((\text{someone } \lambda \langle q, y \rangle. (y \langle c, \lambda c'.(\text{left } \langle c', q \rangle) \rangle))).\end{aligned}$$

Next step:

- ▶ $\llbracket \exists \rrbracket = \text{Value}$
- ▶ $\llbracket \exists \rrbracket = \text{Computation}$

7.2. Lifting for \exists, \forall

Source language

someone	$s/(np \setminus s)$	$(e \rightarrow t) \rightarrow t$	\exists
everyone	$s/(np \setminus s)$	$(e \rightarrow t) \rightarrow t$	\forall

Target language

$$\lceil (s \oslash s) \odot np \rceil = R^{\lceil s \rceil \times R^{\lceil s \rceil}} \times \lceil np \rceil$$

$$\lfloor (s \oslash s) \odot np \rfloor = R^{R^{\lfloor np \rfloor} \times R^{\lfloor s \rfloor \times R^{\lfloor s \rfloor}}}$$

CBV Under cbv, we look for a solution for the type transition

$$\exists^{(e \rightarrow t) \rightarrow t} \xrightarrow[\lambda \rightarrow]{} \lceil (s \oslash s) \odot np \rceil$$

$\lceil (s \oslash s) \odot np \rceil$ is a pair, with the np value we would like \exists to bind as the second coordinate. There is no type transition that can effect this.

7.3. Lifting for \exists, \forall : call-by-name

Under cbn, we look for a type transition of the form

$$\exists^{(e \rightarrow t) \rightarrow t} \xrightarrow{\llbracket \exists \rrbracket}_{\lambda \rightarrow} \lfloor (s \oslash s) \odot np \rfloor$$

$$\lfloor (s \oslash s) \odot np \rfloor = R^{\textcolor{red}{R^{\lfloor np \rfloor}} \times R^{\lfloor s \rfloor} \times \textcolor{red}{R^{\lfloor s \rfloor}}}$$

This time, there is a solution.

$$\llbracket \text{someone} \rrbracket = \lambda Q. (\exists \lambda x. (Q \langle \lambda k. (k x), \lambda \langle \textcolor{brown}{c}, \textcolor{red}{p} \rangle. (p c) \rangle))$$

$$\lambda k. (k x) : R^{\lfloor np \rfloor} = \textcolor{red}{C}_{np}$$

$$\text{LIFT} = \lambda \langle \textcolor{brown}{c}, \textcolor{red}{p} \rangle. (p c) : R^{\lfloor s \rfloor \times \textcolor{red}{R^{\lfloor s \rfloor}}}$$

7.4. Connection to Montague interpretations: CPS

1. $\lambda c.((\text{someone } \lambda \langle q, y \rangle. (y \langle c, \lambda c'. (\text{left } \langle c', q \rangle) \rangle)))$
2. $\lambda c.((\lambda Q. (\exists \lambda x. (Q \langle \lambda k. k \ x, \text{LIFT} \rangle))) \lambda \langle q, y \rangle. (y \langle c, \lambda c'. (\text{left } \langle c', q \rangle) \rangle))$
3. $\lambda c.(\exists \lambda x. (\lambda \langle \text{q}, \text{y} \rangle. (\text{y} \langle c, \lambda c'. (\text{left } \langle c', \text{q} \rangle) \rangle) \langle \lambda k. k \ x, \text{LIFT} \rangle)))$
4. $\lambda c.(\exists \lambda x. (\text{LIFT } \langle c, \lambda c'. (\text{left } \langle c', (\lambda k. k \ x) \rangle) \rangle))$
5. $\lambda c.(\exists \lambda x. ((\text{LIFT } \langle c'', p \rangle. (p \ c'')) \langle c, \lambda c'. (\text{left } \langle c', (\lambda k. k \ x) \rangle) \rangle))$
6. $\lambda c.(\exists \lambda x. ((\lambda c'. (\text{left } \langle c', (\lambda k. k \ x) \rangle)) \ c))$
7. $\lambda c.(\exists \lambda x. ((\text{left } \langle c, (\lambda k. k \ x) \rangle)))$
8. $\lambda c.(\exists \lambda x. ((\lambda \langle c', q \rangle. (q \ \lambda x'. (c' (\text{left } x') \rangle)) \langle c, (\lambda k. k \ x) \rangle)))$
9. $\lambda c.(\exists \lambda x. ((\lambda k. k \ x) \ \lambda x'. (c (\text{left } x') \rangle)))$
10. $\lambda c.(\exists \lambda x. (c (\text{left } x)))$

8. Scope ambiguity, local

Eg., “Everyone teases someone” ($[QP_1 [TV QP_2]]$)

Recall, $[NP [TV NP]]: \mu\alpha.(TV * ((x \times \alpha) \times y))$ ($= M : C_S$)

1. $K_{OBJ}: \tilde{\mu}y.(M \succ \beta_2) * \gamma_2$ ($= K_2$)

γ_2 is the $(s \oslash s)$ of QP_2

2. $C_s: \mu\beta_2.(QP_2 * (\gamma_2 \oslash K_2))$ ($= M'$)

► Obtained by: Co-application, Shift, Co-abstraction, Shift

► Corresponds to: $[np [TV QP_2]]$

1. $K_{SUBJ}: \tilde{\mu}x.(M' \succ \beta_1) * \gamma_1$ ($= K_1$)

γ_1 is the $(s \oslash s)$ of QP_1

2. $C_s: \mu\beta_1.(QP_1 * (\gamma_1 \oslash K_1))$

► Same pattern of rules. Corresponds to: $[QP_1 [TV QP_2]]$ ($QP_1 > QP_2$)

1. $K_{\text{SUBJ}}: \tilde{\mu}y.(M \succ \beta_1) * \gamma_1 (= K_1)$

γ_2 is the $(s \oslash s)$ of QP_2

2. $C_s: \mu\beta_1.(QP_1 * (\gamma_1 \oslash K_1)) (= M')$

► Same pattern of rules. Corresponds to: $[QP_1 \ [TV \ np]]$

1. $K_{\text{OBJ}}: \tilde{\mu}x.(M' \succ \beta_2) * \gamma_2 (= K_2)$

γ_2 is the $(s \oslash s)$ of QP_2

2. $C_s: \mu\beta_2.(QP_2 * (\gamma_2 \oslash K_2))$

► Same pattern of rules. Corresponds to: $[QP_1 \ [TV \ QP_2]] \ (QP_2 > QP_1)$

8.1. Ambiguity: CPS in CBN

From CPS in CBN we obtain the two readings:

$$\begin{aligned} & \lambda c. (\llbracket \text{evr} \rrbracket \lambda \langle q, y \rangle. (\llbracket \text{sm} \rrbracket \lambda \langle p, z \rangle. (y \langle c, \lambda c'. (z \langle c', \lambda c''. (\llbracket \text{teases} \rrbracket \langle p, \langle c'', q \rangle) \rangle) \rangle))) \rangle)) \\ & \lambda c. (\llbracket \text{sm} \rrbracket \lambda \langle p, z \rangle. (\llbracket \text{evr} \rrbracket \lambda \langle q, y \rangle. (z \langle c, \lambda c'. (y \langle c', \lambda c''. (\llbracket \text{teases} \rrbracket \langle p, \langle c'', q \rangle) \rangle) \rangle))) \rangle)) \end{aligned}$$

Substituting the definitions for $\llbracket \cdot \rrbracket$, these reduce to:

$$\lambda c. (\forall \lambda x. (\exists \lambda y. (c ((\text{teases } y) x))))$$

$$\lambda c. (\exists \lambda y. (\forall \lambda x. (c ((\text{teases } y) x))))$$

9. Step by step

1. $\lambda c.(\llbracket \text{evr} \rrbracket \lambda \langle q, y \rangle.(\llbracket \text{sm} \rrbracket \lambda \langle p, z \rangle.(y \langle c, \lambda c'.(z \langle c', \lambda c''.(\llbracket \text{teases} \rrbracket \langle p, \langle c'', q \rangle) \rangle) \rangle)))$
2. $\dots \lambda \langle q, \langle c', q' \rangle \rangle. (q' \lambda x. (q \lambda y. (c' (\text{teases } y) x))) \langle p, \langle c'', q \rangle \rangle)$
3. $\dots (q \lambda x. (p \lambda y. (c'' (\text{teases } y) x))) := M$
4. $\dots (\lambda Q. (\exists \lambda x. (Q \langle \lambda k. k \ x, \text{LIFT} \rangle)) \lambda \langle p, z \rangle. (y \langle c, \lambda c'. (z \langle c', \lambda c''. (M) \rangle) \rangle))$
5. $\dots (\exists \lambda x. ((\lambda \langle p, z \rangle. (y \langle c, \lambda c'. (z \langle c', \lambda c''. (M) \rangle) \rangle) \rangle) \langle \lambda k. k \ x, \text{LIFT} \rangle))$
6. $\dots (\exists \lambda x'. (((y \langle c, \lambda c'. (\text{LIFT} \langle c', \lambda c''. (q \lambda x. (\lambda k. k \ x') \lambda y. (c'' (\text{teases } y) x))) \rangle) \rangle)))$
7. $\dots (\exists \lambda x'. (((y \langle c, \lambda c'. ((\lambda \langle c, p \rangle. (p \ c)) \langle c', \lambda c''. (q \lambda x. (c'' (\text{teases } x') x)) \rangle) \rangle) \rangle)))$
8. $\dots (\exists \lambda x'. (y \langle c, \lambda c'. (q \lambda x. (\textcolor{brown}{c'} (\text{teases } x') x)) \rangle) \rangle) := N$
9. $\lambda c. (\llbracket \text{ev} \rrbracket \lambda \langle q, y' \rangle. (\exists \lambda x'. (y \langle c, \lambda c'. (q \lambda x. (\textcolor{brown}{c'} (\text{teases } x') x)) \rangle) \rangle))$

10. $\lambda c.(\lambda Q.(\forall \lambda x''.(Q \langle \lambda k.k\ x'', \text{LIFT} \rangle)) \ \lambda \langle q, y' \rangle.(N))$
11. $\lambda c.(\forall \lambda x''.(\lambda \langle q, y' \rangle.(\exists \lambda x'.(y' \langle c, \lambda c'.(q \ \lambda x.(c' (\text{teases } x')\ x)))))) \ \langle \lambda k.k\ x'', \text{LIFT} \rangle)$
12. $\lambda c.(\forall \lambda x''.(\exists \lambda x'.(\lambda \langle c, p \rangle.(p\ c) \ \langle c, \lambda c'.(q\ (c' (\text{teases } x')\ x'')))))$
13. $\lambda c.(\forall \lambda x''.(\exists \lambda x'.(q\ (c\ (\text{teases } x')\ x'')))))$

Note the role played by the combinator $\lambda \langle c, p \rangle.(p\ c)$. It corresponds to $(s \oslash s)$ in the derivation.

The unfolding of the other reading proceed in a similar way.

9.1. What if ...

we use the other term for “teases”?

to be continued ...

10. What have you learned today

- ▶ Montague-style and CPS semantics have different interpretation domains
- ▶ We have studied mappings to lift the interpretations of the Montagovian source language to the CPS level
- ▶ Such mappings are not guaranteed to exist, in particular
 - ▶ the constants \exists, \forall have a CPS lifted version under cbn, not under cbv
- ▶ We have started analysis of scope construal phenomena, addressing the Lambek calculus problems of
 - ▶ type uniformity
 - ▶ scope flexibility

Open question How do we impose island constraints on non-local scoping? We address this question in Friday's lecture.

Ambiguity: SUB wide scope

$$\begin{array}{c} \vdots \\ \hline np \circ ((np \setminus s) / np \circ np) \longrightarrow s \quad s \longrightarrow s \\ \hline np \circ ((np \setminus s) / np \circ np) \longrightarrow (s \otimes s) \circ s \\ \hline np \circ ((np \setminus s) / np \circ np) \longrightarrow (s \otimes s) \circ s \\ \hline np \circ ((np \setminus s) / np \circ (s \otimes s) \oslash np) \longrightarrow s \\ \hline np \circ ((np \setminus s) / np \circ (s \otimes s) \oslash np) \longrightarrow s \quad s \longrightarrow s \\ \hline np \circ ((np \setminus s) / np \circ (s \otimes s) \oslash np) \longrightarrow (s \otimes s) \circ s \\ \hline np \circ ((np \setminus s) / np \circ (s \otimes s) \oslash np) \longrightarrow (s \otimes s) \circ s \\ \hline (s \otimes s) \otimes np \circ ((np \setminus s) / np \circ (s \otimes s) \oslash np) \longrightarrow s \\ \hline (s \otimes s) \otimes np \circ ((np \setminus s) / np \circ (s \otimes s) \oslash np) \longrightarrow s \end{array}$$

Ambiguity: OBJ wide scope

$$\begin{array}{c} \vdots \\ \hline np \circ ((np \setminus s) / np \circ np) \longrightarrow s \quad s \longrightarrow s \\ \hline np \circ ((np \setminus s) / np \circ np) \longrightarrow (s \oslash s) \circ s \\ \hline np \circ ((np \setminus s) / np \circ np) \longrightarrow (s \oslash s) \circ np \\ \hline (s \oslash s) \otimes np \circ ((np \setminus s) / np \circ np) \longrightarrow s \\ \hline (s \oslash s) \otimes np \circ ((np \setminus s) / np \circ np) \longrightarrow s \quad s \longrightarrow s \\ \hline (s \oslash s) \otimes np \circ ((np \setminus s) / np \circ np) \longrightarrow (s \oslash s) \circ s \\ \hline (s \oslash s) \otimes np \circ ((np \setminus s) / np \circ np \circ (s \oslash s) \oslash np) \longrightarrow s \\ \hline (s \oslash s) \otimes np \circ ((np \setminus s) / np \circ (s \oslash s) \oslash np) \longrightarrow s \end{array}$$