Typing rules: natural deduction versus sequent derivations

Barker (2001) gives CPS interpretation for terms MN. In natural deduction (N.D.) derivations, a term MN of type B is the conclusion of an implication elimination inference with premise terms N of type A and M of type $A\backslash B$ (or B/A, because the Curry-Howard isomorphism has been weakened to a homomorphism).

The typing rules in Bernardi & Moortgat (2007) are for sequent derivations. In a sequent derivation, from a premise term N of type A and a premise coterm K of type B, one obtains conclusion coterms $N \ltimes K$ of type $A\backslash B$ or $K \rtimes N$ of type B/A. Here, the function type is assigned the complex term.

To compare the two approaches, one can use the standard translation from N.D. to sequent derivations, as discussed in Curien & Herbelin (2000), sections 2 and 3. There is a choice between a left-to-right regime $\cdot^{>}$ and a right-to-left regime $\cdot^{<}$, corresponding to call-by-name and call-by-value reduction respectively. We give the clauses for $M : A\backslash B$; the B/A case is symmetric (with $\rtimes$ instead of $\ltimes$).

$$\begin{array}{rcl} (MN)^{>} & = & \mu\alpha.(M^{>} * (N^{>} \ltimes \alpha)) \\ (MN)^{<} & = & \mu\alpha.(N^{<} * \widetilde{\mu}x.(M^{<} * (x \ltimes \alpha))) \end{array}$$

The CPS interpretation, following B&M (15) and (16), is as follows.

$$\begin{array}{rcl} \lceil (MN)^{>} \rceil & = & \lambda k.(\lceil M^{>} \rceil \ \lambda m.(\lceil N^{>} \rceil \ \lambda n.(m \ \langle n, k \rangle))) \\ \lceil (MN)^{<} \rceil & = & \lambda k.(\lceil N^{<} \rceil \ \lambda n.(\lceil M^{<} \rceil \ \lambda m.(m \ \langle n, k \rangle))) \end{array}$$

Compare the two evaluations in B. For B&M and B, n is a value of type A, and k a continuation of type B. For B&M, m is a value of type $A\backslash B$, i.e. a function from A values to B computations. For B, m is a function from A values to B values.

$$\begin{array}{rcl} \overline{(MN)} & = & \lambda k.(\overline{M} \ \lambda m.(\overline{N} \ \lambda n.k(mn))) \\ \overline{(MN)} & = & \lambda k.(\overline{N} \ \lambda n.(\overline{M} \ \lambda m.k(mn))) \end{array}$$

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