Symmetric categorial grammar

Friday

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Abstract

We study unary residuated and Galois connected operators in the symmetric setting of Lambek-Grishin calculus.

Contents

1. Plan for today

- Residuation and Galois connections in logic and algebra
- Order-preserving and order-reversing modalities
- Decidability
- Completeness for the relational semantics
- Curry-Howard interpretation (work in progress . . .)
- Linguistic applications: islands and scope delimitation

2. Residuation and Galois connections

Dunn 1999 gives a nice overview of the role played by residuation and Galois connection in algebra and logic.

Ordered sets $(X, \leq)$, $(Y, \leq')$ with mappings

$$f : X \longrightarrow Y \qquad g : Y \longrightarrow X$$

The defining biconditionals for residuated pairs (rp), Galois connected pairs (gc), dual rp (drp) and dual Galois gc (dgc) are given below.

rp	$fx \leq' y$	iff	$x \leq gy$
drp	$y \leq' fx$	iff	$gy \leq x$
gc	$y \leq' fx$	iff	$x \leq gy$
dgc	$fx \leq' y$	iff	$gy \leq x$

3. Vocabulary

iff	$x \leq gy$	$gy \leq x$
$fx \leq' y$	rp	dcg
$y \leq' fx$	gc	drp

$$
\begin{array}{lllllllll}
A & ::= & \Diamond A & \mid & \Box A & \mid & A^{\mathbf{1}} & \mid & {}^{\mathbf{1}}A \\
 & & {}^{\mathbf{0}}A & \mid & A^{\mathbf{0}} & \mid & \blacksquare A & \mid & \blacklozenge A
\end{array}
$$

4. Monotonicity, compositions

An equivalent characterization of (d)rp, (d)gc is in terms of the tonicity properties of f, g and their compositions.

Compositions:

	$1_X \leq gf$	$gf \leq 1_X$
$fg \leq' 1_Y$	rp	dgc
$1_Y \leq' fg$	gc	drp

Toniciy:

- (d)rp: f, g are order-preserving ($\Diamond$, $\Box$, $\blacksquare$, $\blacklozenge$)
- (d)gc: f, g are order-reversing ($^{\mathbf{0}}$, $^{\mathbf{0}}$, $^{\mathbf{1}}$, $^{\mathbf{1}}$)

5. Compare: binary and unary operators

$$\frac{B \circ \Gamma \longrightarrow \Delta[A]}{\Gamma \longrightarrow \Delta[B\backslash A]}\ (\backslash\mathsf{R}) \qquad \frac{\langle \Gamma \rangle \longrightarrow \Delta[A]}{\Gamma \longrightarrow \Delta[\Box A]}\ (\Box\mathsf{R})$$

$$\frac{\Delta \longrightarrow B \quad \Gamma[A] \longrightarrow \Delta'}{\Gamma[\Delta \circ B\backslash A] \longrightarrow \Delta'}\ (\backslash\mathsf{L}) \qquad \frac{\Gamma[A] \longrightarrow \Delta'}{\Gamma[\langle \Box A \rangle] \longrightarrow \Delta'}\ (\Box\mathsf{L})$$

$B \otimes B\backslash A \leq A$ $\qquad\qquad$ $\Diamond\Box A \leq A$

$$\frac{\Gamma[D] \longrightarrow \Delta'}{\Gamma[C] \longrightarrow \Delta'} \qquad \text{if } C \leq D$$

6. (Dual) residuation principles

For decidable proof search, we extend our combinator system with the shifting and monotonicity rules for the unary operator cases.

The unary residuated $\Diamond, \Box$ and their duals have the same rules: a cobox is a diamond, a codiamond a box. Cf $\langle F \rangle$ versus $\langle P \rangle$ in temporal logic. Cf Anna Chernilovskaya, Reader Ch 5.

$$\frac{f : \Diamond A \longrightarrow B}{\mathsf{rp}\ f : A \longrightarrow \Box B} \qquad \frac{f : \blacklozenge A \longrightarrow B}{\mathsf{drp}\ f : A \longrightarrow \blacksquare B}$$

Monotonicity principles

$$\frac{A \longrightarrow B}{\Diamond A \longrightarrow \Diamond B} \qquad \frac{A \longrightarrow B}{\blacklozenge A \longrightarrow \blacklozenge B} \qquad \frac{A \longrightarrow B}{\Box A \longrightarrow \Box B} \qquad \frac{A \longrightarrow B}{\blacksquare A \longrightarrow \blacksquare B}$$

Symmetries

$$\begin{array}{rclcrcl}
(\Diamond A)^{\bowtie} & = & \blacklozenge(A^{\bowtie}) & \qquad & (\Box A)^{\bowtie} & = & \blacksquare(A^{\bowtie}) \\
(\Diamond A)^{\infty} & = & \blacksquare(A^{\infty}) & \qquad & (\Box A)^{\infty} & = & \blacklozenge(A^{\infty})
\end{array}$$

7. Grishin interaction principles

The Grishin interactions extend to the unary vocabulary.

$$\blacklozenge A \otimes B \rightarrow \blacklozenge(A \otimes B) \quad A \otimes \blacklozenge B \rightarrow \blacklozenge(A \otimes B) \quad \Diamond\blacklozenge A \rightarrow \blacklozenge\Diamond A$$

Equivalently, there are the dual forms:

$$\Box(A \oplus B) \rightarrow \Box A \oplus B \qquad \Box(A \oplus B) \rightarrow A \oplus \Box B \qquad \Box\blacksquare A \rightarrow \blacksquare\Box A$$

For decidable proof search, we put them in rule form (compiling away the use of transitivity):

$$\frac{\blacklozenge(A \otimes B) \rightarrow C}{\blacklozenge A \otimes B \rightarrow C}\ R1 \qquad \frac{\blacklozenge(A \otimes B) \rightarrow C}{A \otimes \blacklozenge B \rightarrow C}\ R2 \qquad \frac{\blacklozenge\Diamond A \rightarrow B}{\Diamond\blacklozenge A \rightarrow B}\ R3$$

8. (Dual) Galois connected operations

Galois principles:

$$\frac{f : B \to A^{\mathbf{0}}}{\mathsf{lg}\ f : A \to {}^{\mathbf{0}}B} \qquad \frac{f : B \to {}^{\mathbf{0}}A}{\mathsf{rg}\ f : A \to B^{\mathbf{0}}}$$

$$\frac{f : A \to B}{{}^{\mathbf{0}}f : {}^{\mathbf{0}}B \to {}^{\mathbf{0}}A} \qquad \frac{f : A \to B}{f^{\mathbf{0}} : B^{\mathbf{0}} \to A^{\mathbf{0}}}$$

Dual Galois principles:

$$\frac{f : B^{\mathbf{1}} \to A}{\mathsf{ldg}\ f : {}^{\mathbf{1}}A \to B} \qquad \frac{f : {}^{\mathbf{1}}B \to A}{\mathsf{rdg}\ f : A^{\mathbf{1}} \to B}$$

$$\frac{f : A \to B}{{}^{\mathbf{1}}f : {}^{\mathbf{1}}B \to {}^{\mathbf{1}}A} \qquad \frac{f : A \to B}{f^{\mathbf{1}} : B^{\mathbf{1}} \to A^{\mathbf{1}}}$$

9. Truth conditions, completeness

$$
\begin{array}{lcl}
x \Vdash \Diamond A & \text{iff} & \exists y.R_{\Diamond}xy \;\wedge\; y \Vdash A \\
x \Vdash \Box A & \text{iff} & \forall y.R_{\Diamond}yx \Rightarrow y \Vdash A \\
x \Vdash \blacklozenge A & \text{iff} & \exists y.R_{\blacklozenge}xy \;\wedge\; y \Vdash A \\
x \Vdash \blacksquare A & \text{iff} & \forall y.R_{\blacklozenge}yx \Rightarrow y \Vdash A
\end{array}
$$

$$
\begin{array}{lcl}
m \Vdash A^{\mathbf{0}} & \text{iff} & \forall m'.(R_0 mm' \Rightarrow m' \nVdash A) \\
m \Vdash {}^{\mathbf{0}}A & \text{iff} & \forall m'.(R_0 m'm \Rightarrow m' \nVdash A) \\
m \Vdash {}^{\mathbf{1}}A & \text{iff} & \exists m'.(R_1 mm' \wedge\; m' \nVdash A) \\
m \Vdash A^{\mathbf{1}} & \text{iff} & \exists m'.(R_1 m'm \wedge\; m' \nVdash A)
\end{array}
$$

Completeness Unary extensions of relational completeness are in Kurtonina & Moortgat (rp), Chernilovskaya 07 (drp), Areces, Bernardi & Moortgat 01 (gc).

10. Curry Howard interpretation

As an example of the requirements to be met, we work out the sequent term labeling for $\Box$.

Extending the term language

$$
\begin{array}{lll}
{}^{\wedge}M \in \mathsf{Term}^{\Box A} & \text{if} & M \in \mathsf{Term}^{A} \\
{}^{\vee}K \in \mathsf{CoTerm}^{\Box A} & \text{if} & K \in \mathsf{CoTerm}^{A}
\end{array}
$$

Sequent rules In addition to the binary punctuation $(- \circ -)$ (structural counterpart of $\otimes$ in the antecedent), we now have angular brackets $\langle - \rangle$ as structural counterpart of $\Diamond$.

$$
\frac{\langle \Gamma \rangle \xrightarrow{M} \Delta[A]}{\Gamma \xrightarrow{{}^{\wedge}M} \Delta[\Box A]}\ (\Box R)
\qquad
\frac{\Gamma[A] \xrightarrow{K} \Delta}{\Gamma[\langle \Box A \rangle] \xrightarrow{{}^{\vee}K} \Delta}\ (\Box L)
$$

11. β equivalence

The two faces of identity give rise to β and η equivalences. Consider $(\Box\beta)$ first.

$$(\Box\beta) \qquad ({}^{\wedge}M * {}^{\vee}K) \longrightarrow (M * K)$$

$(\Box\beta)$ is the image of the following transformation on sequent proofs.

$$\dfrac{\dfrac{\langle\Gamma\rangle \xrightarrow{M} A}{\Gamma \xrightarrow{{}^{\wedge}M} \Box A}(\Box R) \qquad \dfrac{A \xrightarrow{K} \Delta}{\langle\Box A\rangle \xrightarrow{{}^{\vee}K} \Delta}(\Box L)}{\langle\Gamma\rangle \xrightarrow{({}^{\wedge}M * {}^{\vee}K)} \Delta}(\text{Cut}) \quad \rightsquigarrow \quad \dfrac{\langle\Gamma\rangle \xrightarrow{M} A \qquad A \xrightarrow{K} \Delta}{\langle\Gamma\rangle \xrightarrow{(M * K)} \Delta}(\text{Cut})$$

12. η equivalence

$$\begin{array}{lccc}
(\Box\beta) & ({}^{\wedge}M * {}^{\vee}K) & \longrightarrow & (M * K) \\
(\Box\eta) & x & \longrightarrow & {}^{\wedge}\mu\alpha.(x * {}^{\vee}\alpha)
\end{array}$$

$(\Box\eta)$ is the image of the following proof transformation.

$$x : \Box A \xrightarrow{x} \Box A \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{A \xrightarrow{\alpha} \alpha : A}{\langle \Box A \rangle \xrightarrow{{}^{\vee}\alpha} \alpha : A}\,(\Box L)}{\langle x : \Box A \rangle \xrightarrow{\mu\alpha.(x * {}^{\vee}\alpha)} A}\,(\leftrightharpoons)}{x : \Box A \xrightarrow{{}^{\wedge}\mu\alpha.(x * {}^{\vee}\alpha)} \Box A}\,(\Box R)$$

Next step CPS interpretation. Before turning to it, let us review the linguistic use of the modalities we have in mind.

13. Delimiting control

Island decoration For non-bridge predicates, we decorate the complement sentence with co-box. Recall that $\Diamond$ doesn't block an embedded QP $(s \oslash s) \obslash np$ to take non-local scope: $\Diamond$ is transparent for the Grishin interactions.

$$\begin{array}{rl} \text{(bridge) thinks} & (np\backslash s)/s \\ \text{(non-bridge) thonks} & (np\backslash s)/\blacklozenge s \end{array}$$

The modal decoration on the complement sentence triggers the matching structural decoration, as in the example below, where *Subj* could be a simple noun phrase or a QP. In the latter case, $\blacklozenge$ blocks non-local scope.

$$np \otimes (((np\backslash s)/\blacklozenge s) \otimes \blacklozenge(\mathit{Subj} \otimes (np\backslash s))) \longrightarrow s$$

Challenge Can we make sense of this semantically? In the cbn setting, this means giving a CPS interpretation for $\blacklozenge^{\infty} = \Box$.

14. Derivation

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{s_3 \vdash s_6 \quad np_5 \vdash np_4}{(s_3 \oslash np_4) \vdash (s_6 \oslash np_5)}\oslash}{s_3 \vdash ((s_6 \oslash np_5) \oplus np_4)}\blacktriangleright'}{\Box s_3 \vdash \Box((s_6 \oslash np_5) \oplus np_4)}\Box \qquad \dfrac{s_7 \vdash s_2 \quad np_1 \vdash np_0}{(s_7 \oslash np_0) \vdash (s_2 \oslash np_1)}\oslash}{(\Box((s_6 \oslash np_5) \oplus np_4) \obslash (s_7 \oslash np_0)) \vdash (\Box s_3 \obslash (s_2 \oslash np_1))}\obslash}{(s_7 \oslash np_0) \vdash (\Box((s_6 \oslash np_5) \oplus np_4) \oplus (\Box s_3 \obslash (s_2 \oslash np_1)))}\blacktriangleleft'}{s_7 \vdash ((\Box((s_6 \oslash np_5) \oplus np_4) \oplus (\Box s_3 \obslash (s_2 \oslash np_1))) \oplus np_0)}\blacktriangleright'
$$

The CPS image of this proof is the term below.

$$\lambda c.(\llbracket \text{thonks} \rrbracket \ \langle \lambda k.(k \ \lambda c'.(\llbracket \text{left} \rrbracket \ \langle c', \llbracket \text{leopold} \rrbracket \rangle)), \langle c, \llbracket \text{molly} \rrbracket \rangle \rangle)$$

After lexical substitution, we would like to associate the derivation with the following interpretation.

$$\lambda c.(c \ ((\text{thonks} \ \lambda c'.(c' \ (\text{left leopold})) \ \text{molly})$$

15. CPS interpretation: types

Types For the domain of interpretation of $\Box A$, we consider two candidates.

$$
\begin{array}{lcl}
(\dagger) \quad \lceil \Box A \rceil & = & R^{R^{\lceil A \rceil} \times R^{R^R} \times R^R} \\
& \cong & (\lceil A \rceil \to ((R \to R) \to R)) \to ((R \to R) \to R)
\end{array}
$$

$$
\begin{array}{lcl}
(\ddagger) \quad \lceil \Box A \rceil & = & R^{R^{\lceil A \rceil}} \\
& \cong & (\lceil A \rceil \to R) \to R
\end{array}
$$

Both options take $\lceil A \rceil$ to a lifted level: R in the case of ($\ddagger$), computation of R in the case of ($\dagger$).

16. CPS interpretation: terms

Consider first the (†) option where $\lceil \Box A \rceil = (\lceil A \rceil \to C_R) \to C_R$.

$$\Box A \qquad \lceil {}^\wedge M \rceil = \lambda k.(k\ \lambda \langle m, c\rangle.(\lceil M \rceil\ \lambda x.(m\ \langle x, c\rangle))) \qquad M : A$$

$$\Box A \qquad \lceil {}^\vee K \rceil = \lambda h.(h\ \langle\ \lambda \langle x, c\rangle.(c\ (\lceil K \rceil\ x)), \mathtt{ID}\ \rangle) \qquad K : A$$

Remark $\lceil {}^\vee K \rceil$ is the reverse of the `SHIFT` construct.

$$\begin{array}{rl} \mathtt{SHIFT} & ((V_A \to C_R) \to C_R) \to (K_A \to R) \\ \mathtt{SWAP\ SHIFT} & K_A \to (((V_A \to C_R) \to C_R) \to R) \end{array}$$

With this interpretation $(\Box\beta)$ is satisfied at the CPS level. But $(\Box\eta)$ is not . . .

$$\lceil ({}^\wedge M * {}^\vee K) \rceil = (\lceil {}^\wedge M \rceil\ \lceil {}^\vee K \rceil) = (\lceil M \rceil\ \lceil K \rceil)$$

17. CPS interpretation (cont'd)

The (‡) option maps an A value to an A computation $(\lceil A \rceil \rightarrow R) \rightarrow R$.

We give the CPS interpretation for the monotonicity rule.

$$\frac{f : A \rightarrow B}{f^{\Box} : \Box A \rightarrow \Box B}$$

$$\lceil (f^{\Box})^{\triangleleft} \rceil \; \beta \quad = \quad \lambda k.(k \; \{\lceil f^{\triangleleft} \rceil \; \lambda y.(\beta \; \lambda c.(c \; y))\})$$

And for the $\overline{\lambda}\mu\tilde{\mu}$ proof terms.

$$\begin{array}{lll} \Box A & \lceil {}^{\wedge} M \rceil = \lambda h.(\lceil M \rceil \; \lambda x.(h \; \lambda c.(c \; x))) & M : A \\ \Box A & \lceil {}^{\vee} K \rceil = \lambda k.(k \; \lceil K \rceil) & K : A \end{array}$$

18. Non-bridge predicates: lexical semantics

Recall the proof term for 'Molly thonks Leopold left' (non-bridge predicate 'thonks').

$$\lambda c.(\llbracket \text{thonks} \rrbracket\ \langle \lambda k.(k\ \lambda c'.(\llbracket \text{left} \rrbracket\ \langle c', \llbracket \text{leopold} \rrbracket \rangle)), \langle c, \llbracket \text{molly} \rrbracket \rangle \rangle)$$

The complement is a $\Box s$ computation, i.e. $((\lfloor s \rfloor \to R) \to R) \to R$, which lowers to $\lfloor s \rfloor \to R$ (s computation) by the combinator below.

$$\begin{array}{ll} \texttt{LOWER} & (((A \to B) \to B) \to C) \to (A \to C) \\ & \lambda h \lambda v.(h\ \lambda c.(c\ v)) \end{array}$$

We then have the following lexical recipe for the non-bridge predicate, resulting in the desired reduced reading.

$$\llbracket \text{thonks} \rrbracket \quad = \quad \lambda \langle p, \langle c, q \rangle \rangle.(q\ \lambda x.(c\ ((\text{thonks}\ (\texttt{LOWER}\ p))\ x)))$$

$$\lambda c.(c\ ((\text{thonks}\ \lambda c'.(c'\ (\text{left leopold})))\ \text{molly}))$$

19. Homework

In ongoing work we pursue this line further to better understand the use of modal decoration in NL semantics (as in Bernardi & Szabolcsi 2007).

- What is the semantic import of a given modal decoration?
- What is the linguistic significance of derivational ambiguities among such decorations?

Example The type transition $\Box\Diamond p \rightarrow \Box\Diamond\Box\Diamond p$ has two distinct readings.

$$\dfrac{\dfrac{\dfrac{\dfrac{p \rightarrow p}{\Diamond p \rightarrow \Diamond p}}{p \rightarrow \Box\Diamond p}}{\Diamond p \rightarrow \Diamond\Box\Diamond p}}{\Box\Diamond p \rightarrow \Box\Diamond\Box\Diamond p} \qquad \dfrac{\dfrac{\dfrac{\dfrac{p \rightarrow p}{\Diamond p \rightarrow \Diamond p}}{\Box\Diamond p \rightarrow \Box\Diamond p}}{\Diamond\Box\Diamond p \rightarrow \Diamond\Box\Diamond p}}{\Box\Diamond p \rightarrow \Box\Diamond\Box\Diamond p}$$

$$((\mathsf{rp}((1_p)^{\Diamond}))^{\Diamond})^{\Box} \qquad \mathsf{rp}((((1_p)^{\Diamond})^{\Box})^{\Diamond})$$

20. More to Explore

Apart from Chapter 5 of the Course Materials, we would like to recommend the following two references for systematic exploration of the landscape of possible type-forming operations.

- Rajeev Goré (1998) gives a comprehensive picture of substructural logics from the Display Logic perspective. There is a link on our References page.
- Richard Moot (2007) 'Proof Nets for Display Logics' develops the theory of Proof Nets for these systems, with a discussion of LTAG simulation via the Grishin interaction postulates. We hope to see his paper on these pages soon!

21. Wrapping up

As an introduction to a general discussion, we summarize the grammatical architecture of **LG**.

- The central component is an algebra of proofs
  - Directionality is part of the tectogrammatical organization.
  - Symmetry and structure-preserving interaction account for dependencies beyond the reach of (**N**)**L**
- The derivations have two interpretations, reflecting the form and meaning dimensions of linguistic signs:
  - Frame semantics provides interpretation for relations of Merge, feature checking, incompatibility, ... $\leadsto$ completeness
  - Curry-Howard semantics relates derivations to instructions for meaning assembly, via a CPS transformation